The equation of continuity for a homogeneous electron beam is

$$\frac{1}{\sigma_b}\frac{dI_b}{dz}+j\omega\rho_b=0. \qquad (5)$$

From (1), (2), (3) and (5), we obtain

$$\frac{dI_b}{dz}=\left(\frac{j\omega I_0}{2u_0V_0}\right)V_b, \qquad (6)$$

where, under the small signal conditions, the ac beam voltage is

$$V_b=\frac{u_0v_b}{\eta}. \qquad (7)$$

The equation of motion for the lossless plasma is

$$j\omega v_p=\eta E, \qquad (8)$$

from which, by using (2) and (4), we obtain

$$I_p=\frac{\sigma_p}{\sigma_b}\cdot\frac{\omega_p^2}{\omega^2}\cdot\frac{1}{\left(1-\frac{\omega_p^2}{\omega^2}\right)}\cdot I_b. \qquad (9)$$

By proper manipulation of (1), (2), (7) and (8), the following equation is obtained:

$$\frac{dV_b}{dz}=\frac{j}{\omega\epsilon_0\sigma_b}\,\frac{1}{\left(1-\frac{\omega_p^2}{\omega^2}\right)}\cdot I_b. \qquad (10)$$

It is noted that (6) is the same equation derived by Bloom and Peter for the case of a modulated beam.[1] Eq. (10) is, however, modified by the factor

$$\left(\frac{1}{1-\frac{\omega_p^2}{\omega^2}}\right),$$

which takes the presence of plasma into account. It may be shown that these two simultaneous differential equations [(6) and (10)] describing the beam-plasma interaction are formally identical with those of the transmission line loaded with lumped resonant circuits, as shown in Fig. 1. We have, from Fig. 1,

$$\frac{dV}{dz}=jXI; \qquad \frac{dI}{dz}=jBV. \qquad (11)$$

There is, therefore, a complete correspondence if line voltage V and line current I correspond to the ac beam voltage V_b and the ac beam current I_b respectively.

The essential feature to be considered here is that the series impedance is capacitive for $\omega>\omega_p$ and inductive for $\omega<\omega_p$. This behavior is very similar to a continuous multicavity klystron or easitron where the resonators are tuned at a frequency such that they present a lossless negative susceptance to the electron beam. It is known in this case of an inductive circuit admittance that there exist increasing waves in the system.[2] For $\omega<\omega_p$, therefore, the beam-plasma system would lead to growing and decaying waves, as has been verified experimentally by several authors.[3,4]

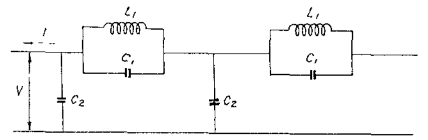

Fig. 1—Line analog of an electron beam in a lossless plasma.

$$C_2=\epsilon_0\sigma_b\left(\frac{2\pi}{\lambda_p}\right)^2 \text{farads}/m \qquad C_1=\epsilon_0\sigma_b \text{ farads}/m$$

$$L_1=\frac{1}{\eta\sigma_b\rho_{0p}} \text{ henrys}/m \qquad \omega_p^2=\frac{1}{L_1C_1}.$$

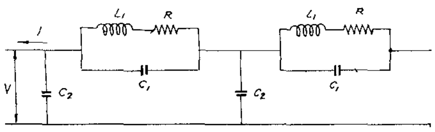

Fig. 2—Line analog of an electron beam in a plasma, with collisions taken into account.

$$R=\nu L_1=\frac{\nu}{\eta\sigma_b\rho_{0p}} \text{ ohms}/m$$

where ν = plasma collision frequency.

Losses in the plasma from collision effects are readily included in the analog line by the introduction of a resistive component in the line series impedance, as shown in Fig. 2.

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Correction to "A Method for Measuring the Directivity of Directional Couplers"

B. C. Yates of the Radio Standards Laboratory, National Bureau of Standards, Boulder, Colo., has brought to the attention of the authors of the above paper[1] errors in some of the equations for error limits. The authors have investigated the difficulty and derived correct equations. If one uses the erroneous expressions, error limits are obtained which are much too conservative.

The equations in question are numbered (18) and (19). The correct expressions are

$$20\log_{10}\frac{1-|\Gamma_{2i}|^2}{1-\left|\frac{\Gamma_{2i}}{K}\right|} \geq \epsilon_{II,1} \geq 20\log_{10}\frac{1-|\Gamma_{2i}|^2}{1+\left|\frac{\Gamma_{2i}}{K}\right|}, \qquad (18)$$

and

$$\epsilon_{II,2}=20\log_{10}\frac{1-|\Gamma_{2i}|^2}{1-\left|\frac{\Gamma_{2i}}{K}\right|^2}. \qquad (19)$$

It should be noted that the approximation $y/2\approx(\Gamma_{2i}/K)$ is no longer needed in the derivations of (18) and (19), and that (19) can represent a correction rather than an error limit if $|\Gamma_{2i}|$ and $|K|$ are known. In order for (18) and (19) to hold,

$$|\Gamma_{2i}|<\frac{1-\frac{1}{|K|}}{1+\frac{1}{|K|}},$$

but the values of $|\Gamma_{2i}|$ and $|K|$ normally encountered will be found to satisfy this inequality.

The graphs of Fig. 5, which were based upon (18) and (19), are no longer correct. However, it was found that, for procedure 1, a sufficiently accurate answer can be obtained by dividing the decibel error limits by 3.

Fig. 5 does not give the correct results for procedure 2. However, it was found that for $|K^2|>10$, (19) can be considered as insensitive to directivity, and equal to 20 $\log_{10}(1-|\Gamma_{2i}|^2)$.

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Multifrequency Microwave Generation Using a Large Capacitance Tunnel Diode

Fundamental, simultaneous oscillation at two discrete microwave frequencies has been experimentally verified using an inexpensive tunnel diode. The diode possesses relatively high junction capacitance of 90 pf (see Fig. 1).

The diode, whose cost is below three dollars, was a 1N3718 and was mounted in an impedance transformation waveguide mount,[1] as illustrated in Fig. 1. In this case the effects of package and junction capacitance are not reduced. The diode package is performing as a cavity resonator, resonating in the X band. The diode was found to oscil-

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